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Stretching Mental Processes: An Overview of and Guide for SFT Applications

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Systems Factorial Technology (SFT) is an advanced non-parametric suite of tools used to analyze the structure of mental processes in cognitive tasks. The major obstacle of adapting SFT to various cognitive tasks is the development of appropriate experimental manipulations to instantiate the double factorial manipulations. This chapter provides an overview of different SFT applications across various psychological modalities and cognitive tasks (visual detection, Stroop, classification and short-term memory search) To facilitate the methodological growth of the SFT applications, a general approach for understanding methodological manipulations is proposed. The integrated view provides the guidelines of how to master the stretching of mental processes within one applicable factorial framework.

Scientific work on decoding cognitive system operations is analogous to reverse engineering. Reverse engineering in software development and electrical circuits design is not strictly focused on learning the exact blueprint of the device, but rather the goal is to design a surrogate device that shows input-output behavior resembling the original. A famous example is the reverse engineering of the original IBM-PC BIOS code that resulted in creation of a completely different code but with operation identical to the original (e.g. Schwarz, 2001). By contrast, the main goal of modelling in cognitive psychology is to learn the exact blueprint of the cognitive system.

To learn a system's blueprint, both cognitive scientists and engineers resort to the analysis of the relationship between inputs and outputs of the system of interest. This procedure is based on probing the system with different input values and observing the system's responses. Unlike engineers, cognitive scientists face a complex problem: even when there is a fixed input to system, the output is stochastic. Thus, the analysis of cognitive activities requires unique tools that can deal with the output variability.

As discussed in the tutorial chapter (Altieri, Fifić, Little, & Yang, 2016), cognitive activities can be organized according to a *mental architecture*, a systematically organized set of mental operations, which entails the fundamental properties of: *processing order* (e.g., serial vs parallel), *stopping rule* (e.g., self-terminating vs exhaustive), *process interdependency* (e.g., independence vs dependence), and *processing capacity* (e.g., limited, unlimited, or super capacity).

For example, to provide evidence about whether processing order is serial or parallel, one must probe the system with inputs such that only the processing order is affected. To ensure that the output will provide meaningful interpretation, two key elements are required: First, the assumption of selective influence, which states that varying the values of an external variable, *X,* should affect only one mental process in the mental architecture. There is no *a priori* way to test the selective influence assumption; however, it is possible to test the resulting output of the system for possible violations of the assumption. (See Townsend, Liu & Zhang, 2016; this volume) for more details on selective influence. Second, the adoption of the factorial design ensures that specific input probes can be generated. Selective influence is important, but it is not sufficient to uncover the true blueprint of the system. To uncover underlying mental architectures consisting of the number of processes, *n*, it is necessary to use the *n* input probes, one for each of the processes of interest. The factorial design allows one to input all of the probes at the same time and examine the output as a function of lengthening different combinations of processes.

In this chapter, we will present a general framework for understanding the methodological manipulations required for using SFT. We will then review the four different SFT applications. These include: (1) visual detection (Townsend & Nozawa, 1995), (2) the Stroop effect (Eidels, 2010), (3) categorization (Fifić, Little, & Nosofsky, 2010; Little, Nosofsky, & Denton, 2011), and (4) memory search (Townsend & Fifić, 2004). To provide better insights into creating a SFT factorial design, these four different studies will be compared at the conceptual level of the fundamental organization of processes. Additionally, the tutorial provides basic instructions on applying SFT to an original study design.

Factorial design: the reverse engineering tool in cognitive psychology

A factorial design is obtained by cross-combining of all the factors' values. Figure 1 displays a two-factorial design in which each factor is represented by a single dimension. The factors form a Cartesian coordinate system (i.e., all combinations of each level of each dimension). In Figure 1, the first dimension is the variable that is assumed to affect the speed of processing of process one. The second dimension is the variable that is assumed to affect speed of processing of process two. Each factor in Figure 1 has two values defined as an external manipulation that either slows down (value of Low) or speeds up (value of High) the processing of each stimulus. A question arises here: what kind of mental architecture underlies the processing of these two dimensions (i.e., in terms of the fundamental properties)?

How does the factorial research design aid the discovery of underlying mental structures?

The factorial design provides two tests:

- *Main effects* or *existence test*: This test provides information as to whether or not a variable had an impact on the mental architecture under investigation. In the example in Figure 1, the main effect of dimension one is achieved by marginalization of dimension two and the comparison of the resulting marginal values of processing speed. If a difference is found, then one can conclude that the manipulation of that dimension affected the process of interest. However, main effects have limited diagnostic power as their analysis cannot provide any interesting information about the fundamental properties (processing order, stopping rule, or interdependency).

- *Interaction effect* or *coexistence test:* Whereas the main effect focuses on one variable at a time, the interaction test analyzes the relationship between both (or several) variables. When applied to the appropriate dependent variable (e.g., the estimated survivor function), the fundamental properties can be successfully diagnosed by the presence of different types of interactions.

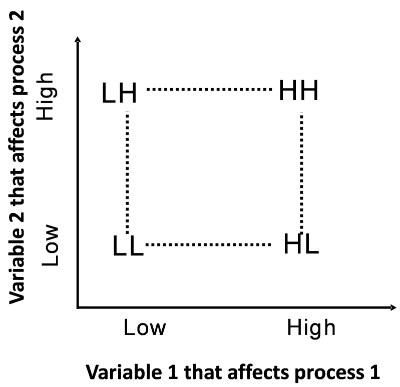


Figure 1: A 2x2 factorial design: Variable 1 (Low, High) x Variable 2 (Low, High). Variable values are orthogonally combined, resulting in the four experimental conditions. The letter position indicates variable (first and second), and the letter L=low, H=High.

Probing the processes: stretching and inserting

Manipulating each of the input factors serves to probe an unknown system in order to reverse engineer that system. Electrical engineers perform reverse engineering by by sending a weak electrical current through the system and observing the response in the output circuit. Similarly, cognitive scientists present an input stimulus and observe the human response. For these stimulus inputs, the dimensions (and their levels) are selected to affect the duration of mental processing. I will denote the unknown process with a capital letter *X*. The experimental manipulation (probe) is denoted with small caps letter *x*. The effect of manipulation *x* on the process *X* is designed to selectively slow down or speed up the process *X* and is denoted as *stretching*. In practice, such an experimental factor could be, for example, the brightness or contrast of a target varied in two levels during a detection task. Lower levels of brightness will lead to a slower detection while higher levels of brightness will lead to faster detection .

The stretching effect is defined as the two-state output difference: probing a system with a low input value x_{low} and then with high input value x_{high} should lead to the output difference measured as response time (RT): $RTx_{low} - RTx_{high}$. The stretching effect is the difference in the RTs that a system takes to process the information at the Low versus High levels. To be able to deal with the inherent output variability, the stretching effect of process *X* is defined as a first-order difference Δ on expected processing time of the process *X* (see also Townsend & Thomas, 1994; Fifić, 2016).

$$\Delta E[t_X;x] = E[t_X;x_{low}] - E[t_X;x_{high}] > 0 \qquad (1)$$

Selective influence operates at the level of mean RTs and is satisfied if the order of expected response time holds: $E[t_X;x_{low}] > E[t_X;x_{high}]$. The unbiased estimator of the true population's expected response times is a simple mean response time value. Thus the inequality could be rewritten in a simpler form as: $RT_{low} > RT_{high}$.

Stretching of two factors and additivity

Testing for factorial interactions employs the mean interaction contrast (MIC) which has its origins in the analysis of variance (ANOVA). To calculate the MIC for the two processes, the second-order difference Δ^2 is derived for two variables, (x, y) each belonging to distinct process X and Y, respectively, within an unknown mental network:

$$\Delta^2 E[t_X, t_Y; x, y] =$$
$$\left(E\left[t_X, t_Y; x_{low}, y_{low}\right] - E\left[t_X, t_Y; x_{low}, y_{high}\right]\right) - \left(E\left[t_X, t_Y; x_{high}, y_{low}\right] - E\left[t_X, t_Y; x_{high}, y_{high}\right]\right) \qquad (2)$$

Each term in (2) represents the expected response time for two joint processes t_X and t_Y combined within the same mental architecture. The stretching manipulation for each process, is manipulated at the low and high level so, for example, the notation (x_{low}, y_{high}) represents an experimental condition in which process X is stretched at the low level and process Y at the high level. The above equation could be written in the form of mean response times for each condition (Altieri et al., 2016, Chapter 1).

$$MIC = (RT_{LL} - RT_{LH}) - (RT_{HL} - RT_{HH}) =$$
$$RT_{LL} - RT_{LH} - RT_{HL} + RT_{HH} \qquad (3)$$

Factorial additivity is revealed when the second order difference is zero, overadditivity is indicted by the positive value, and underadditivity by the negative value. As discussed in the tutorial chapter (Altieri, et al., 2016), in addition to this mean level contrast, SFT provides a stronger statistic using survivor functions. Both the MIC and SIC statistics provide sufficient information to explore the fundamental properties of the two mental process.

Implementing the Systems Factorial Technology

For novices in using SFT technology, one of the main concern is how to implement the methodology in an examined cognitive task. In the following we describe three important steps to implementing SFT. These steps include:

(1) Defining a processing model of interest with respect to its fundamental properties: processing order, stopping rule, and process interdependency, and identifying the processes within the model that will be explored.

(2) Stretching the durations of the processes by using new external variables that selectively affect the processes of interest.

(3) Collecting multiple trials RT data, analyzing the corresponding response distributions, and interpreting the results using the inference tools provided from SFT, and then using the inferences either to falsify or to validate the candidate models.

Step 1: Modelling and identification of mental processes

In the first step, we will review the process of identification of mental processes within proposed cognitive models and identify the fundamental components of the competing cognitive models that could be validated by SFT. To facilitate this overview, we will briefly summarize some of the applications which are reviewed in this volume but at a broad general level.

Stimulus detection and redundant target task

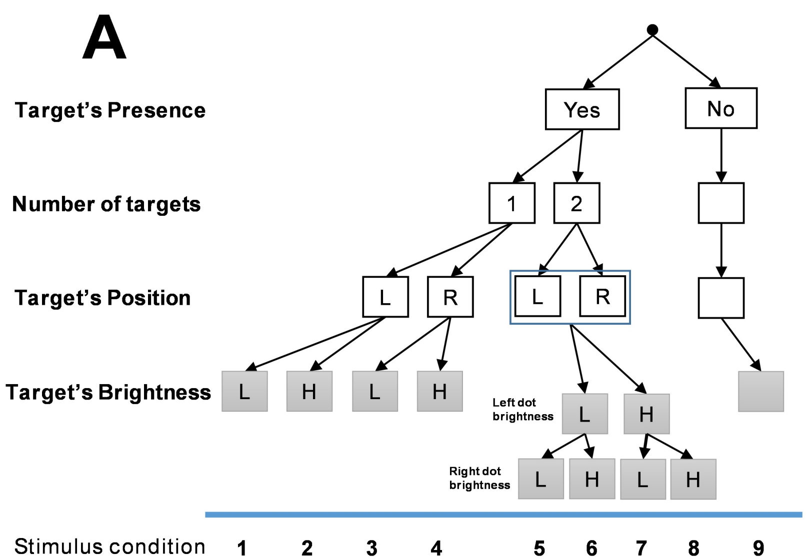
As discussed in the tutorial chapter (Altieri, et al., 2016), SFT was initially applied to the question of stimulus detection (see also Townsend & Nozawa 1995). In the task either one or two dots are displayed in either the left or right visual field. The task is to detect any dot and make a decision by pressing a response button.

Research Design. The original study had three factors, showed in white boxes in Figure 2A. The factor of target presence indicates whether a target dot was presented or not in a display (Yes or No). The target absent conditions are indicated by the empty box; the factor of number of targets indicates how many target stimuli were presented in the display (1, 2, or none –empty box); The factor of target position indicates whether a target was presented in the left (L) or right (R) visual field, and in the two-target conditions both positions were used (two boxes joined by a frame). Conceptually, what SFT adds to the original study is represented in Figure 2A using the shaded boxes. The factor of target brightness was manipulated at the low (L) and high (H) brightness levels. The magnitude of the targets brightness is carefully selected so that its manipulation either slows down or speeds up the rate of processing of each processing unit (i.e., the left or right dot). The diagnostic SFT conditions –HH, HL, LH, and LL, are displayed at the very bottom of the factorial tree (numbered as 5, 6, 7, and 8). These conditions are all target-present conditions with two dots. Each dot is manipulated at the high and low level of brightness and factorially combined with the brightness level of the other simultaneously presented dot. The four conditions are also displayed in the Figure 2B, which is another view of the factorial tree in Figure 2A, obtained by renaming some of the variables from the panel A. Panel B shows the two orthogonal dimension variables, the brightness of a target dot in the left and right locations. The two dimensions in panels B and C combine information about the target dot position, target dot brightness, and the number of target dots within the full factorial design. Panel C shows the actual study trials, defined in the panel B.

It is important to note that although the stimulus conditions (1, 2, 3, and 4) are factorially combined with the target brightness level, they do not permit the SFT analysis, since only one target dot is presented in a display. Nonetheless, these conditions can be used to conduct the additional contrast test that could be used to validate the SFT analyses, as will be described later (e.g. Fific, Little, & Nosofsky, 2010).

Defining a model. The typical finding in this task is that detection time decreases as the number of targets increases. Using a parallel processing model, several alternative explanations have been offered for this result. According to the first, RT decreases due to so-called statistical facilitation (Todd, 1912; Grice, Canham & Boroughs, 1984; Diederich & Colonius, 1987). As the number of possible target increases, the response time decreases as the probability to detect any target item increases. This is a statistical effect for systems characterized as parallel self-terminating mental architectures. The alternative explanation is based on the idea that detection of a single target depends on detection of other targets. According to this approach, there is facilitatory effect between target detectors within a parallel architecture. One version of that model is the coactive model (Miller, 1982; Townsend & Nozawa, 1995; Colonius, 1990). This particular coactive architecture is probably best described as a parallel facilitatory system, meaning that the stopping rule is not terminating, but rather the system "pools" the evidence coming from separate channels into one channel.

The most important distinction between the two approaches is focused on the role of the fundamental properties of process interdependence. The coactive model assumes that the processes of interest are interdependent while the statistical summation model assumes that the processes are independent. The statistical facilitation model which assumes a parallel race has a strictly positive SIC function. The coactive model assumes parallel interdependent processes and has an S-shaped SIC function (with a smaller negative part then positive part). The use of this task in the study of attention is discussed by Yang (2016; this volume).



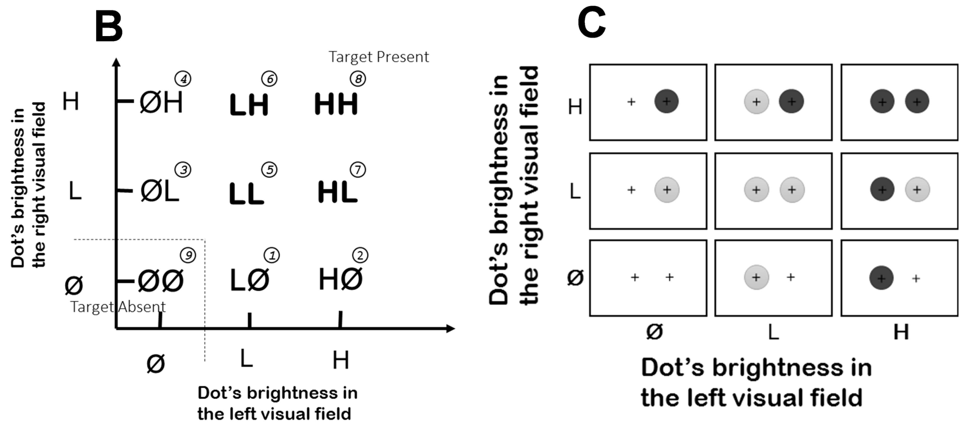


Figure 2: Stimulus detection in the redundant target task. **(A)** The factorial tree represents the factors on left and the factor levels in boxes on right. The white boxes indicate the original study factorial design prior to the SFT modification. The shaded boxes indicate the added conditions after SFT process stretching has been applied. The double-factorial conditions are shown at the tree's bottom, in the form of another two-branch factorial tree made of two factors (left and right dot brightness). **(B)** The corresponding stimulus configuration space is defined by the two continuous dimensions: brightness of a target at either left or right stimulus fields. Zero symbol ø indicates the absence of the stimulus intensity. The double-factorial design is presented in the upper right corner (HH, HL, LH, and LL). The piecewise linear function separates the two response mapping regions: target present - which uses a disjoint rule (OR) to classify stimuli, and target absent – which uses a conjoint stimulus classification rule (AND). The encircled numbers indicate the stimulus conditions denoted in the factorial tree panel A, bottom line. **(C)** Actual stimulus condition realizations of the configuration presented in panel B.

The Stroop task

The Stroop effect demonstrates the inability of humans to selectively attend to one source of information, while trying to ignore a second source of information (Stroop, 1935; also reviewed in more detail in this volume (Algom, Fitousi, & Eidels, 2016)). A focused attention task is used to demonstrate the Stroop effect: subjects were asked to respond as to whether a word stimulus is displayed in, e.g., red and to ignore other sources of information such as word meaning or the presence of other colors than red. The Stroop effect is operationalized as a performance decrement for an incongruent condition ("Green" word in red color) compared to a congruent condition ("Red" word in red color). In a dividend attention version of the Stroop task (Eidels, Townsend & Algom, 2009) a word is displayed, and the subject is asked to detect whether the word contains any information about a target color either in the word or the color of the font. The question in both tasks is how the information, from each source, word and color, is combined to make a decision?

Research Design: The study used in Eidels et al. (2009) had three factors represented in the white boxes of Figure 3A. The factor of target presence indicates whether the red information was presented or not (Red or No Red). There are two targets of different modality: red as a color or red as a word. The target-absent conditions in which no red information is present are indicated by the empty box. The factor of number of targets indicates how many target stimuli were presented in the display (color red and/or word "Red"). The target modality indicates whether the target was color Red or a word "Red", while in the two-target conditions both modalities were used (two boxes joined by a frame).

The SFT addition to the original study is showed in Figure 3A, as the shaded boxes. The factor of target saliency was factorially manipulated and defined as either low (L) and high (H) saliency depending on the level of the manipulation. The magnitude of the saliency manipulation (the stretching effect) is carefully selected so that it either slows down or speeds up the rate of processing of each target modality: either the readability of the words' letters or the magnitude of the red color saturation. Readability was manipulated by using fonts which allowed for either faster or slower word recognition. Color identification was achieved by manipulating the amount of color saturation. The double factorial conditions (HH, HL, LH, and LL) are displayed at the very bottom of the factorial tree – these are achieved by factorial combination of the word and color saliency-- and are numbered as the 5, 6, 7, and 8 conditions. The four conditions are also displayed in the Figure 3B, which is obtained by collapsing and combining of some factors from the factorial tree in panel A. Panel B uses the two continuous orthogonal dimension variables, the color and word saliency that combine information about target presence, number of targets, target modality, and target's saliency, within a fully factorial design. The experimental condition values (1-9) shown in panel A, are also displayed in panel

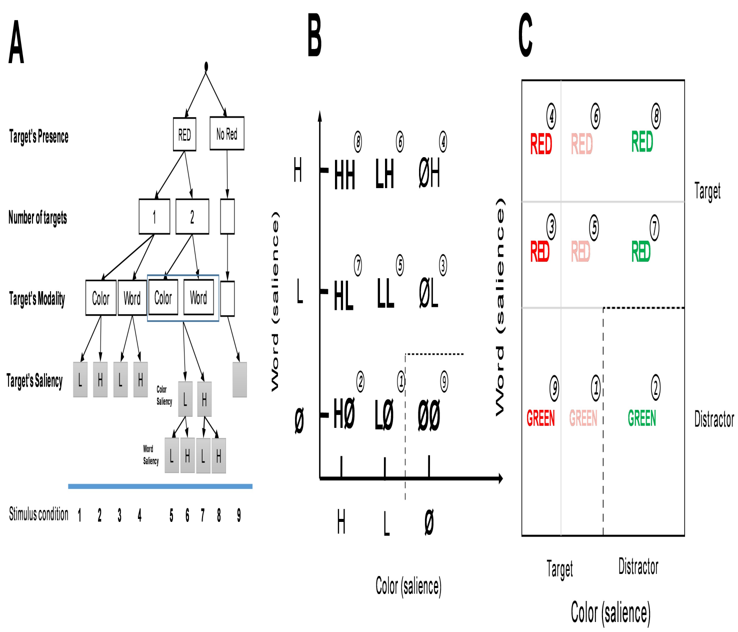
B. The panel C shows the actual trials, defined in the panel B.

Defining a model. There is shared agreement between many researchers that the Stroop effect is due to some form of antagonistic competition between processes analyzing the color and word information (e.g. Cohen, Dunbar, & McClelland, 1990; Logan, 1980; MacLeod & Dunbar, 1988; Phaf, van der Heiden, & Hudson, 1990; Roelofs, 2003; Virzi & Egeth, 1985; Zhang, Zhang, & Kornblum, 1999). An overview of the literature reveals a general consensus that the organization of mental processes underlying the Stroop effect conforms to a interactive parallel architecture.

One alternative approach proposes that the parallel minimum time race model architecture can account for the Stroop effect (Algom, Dekel, & Pansky, 1996; Dishon-Berkovits & Algom, 2000; Melara & Algom, 2003; Melara & Mounts, 1993; Sabri, Melara, & Algom, 2001; Eidels, et al., 2010). In contrast to the class of models adopted by the first approach, the proposal is that the processes analyzing sources of information (color and words) are independent.

Without belaboring the details, it is clear that the most important distinction between the two approaches is focused on the role of the fundamental properties of process interdependence. Both approaches assume that the color and word meaning are organized in parallel. The difference is that the first approach assumes that the two processes inhibit each other while the second approach assumes that the two processes are independent. When both models are tested at the level of the mean RT, the analysis is inconclusive since both models predict slower mean RTs for the incongruent condition then for the congruent condition.

The two models can be distinguished at the RT-distribution level using the SIC . The parallel interactive model predicts an S-shaped SIC function with smaller negative part then positive part; by contrast, the parallel race model predicts a strictly positive SIC function (Townsend & Nozawa, 1995). Further contrasting evidence can be found by measuring the capacity coefficient (see Eidels, Houpt, Altieri, Pei & Townsend, 2011). Further details about this task are discussed in Algom, Fitousi, and Eidels (2016; this volume).



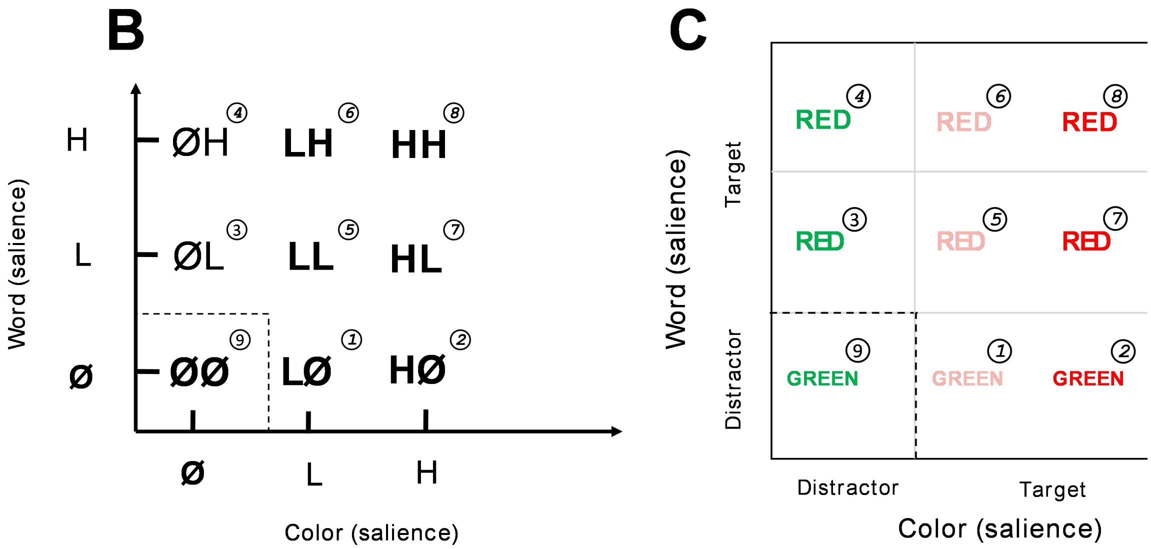


Figure 3: The Stroop task. **(A)** The factorial tree represents the factors on left and the factor levels in boxes on right. The white boxes indicate the original study factorial design prior to the SFT modification. The shaded boxes indicate the added conditions after SFT process stretching has been applied. The double-factorial conditions are shown at the tree's bottom as a two-branch factorial tree made of two factors (Color and Word saliency). **(B)** The corresponding stimulus configuration space is defined by the two continuous dimensions: Color saliency and Word saliency. The symbol ø indicates the absence of the saliency magnitude. The "øø" condition has the zero value of redness and the zero value of alphanumeric "Red" to indicate meaning of red. The double-factorial design is presented in the upper left corner (HH, HL, LH, and LL). The piecewise linear function separates the two response mapping regions: target present - which uses a disjoint rule (OR) to classify stimuli, and target absent – which uses a conjoint stimulus classification rule (AND). The OR region is defined by presence of any of the targets indicating "Red", while the AND region is defined by the absence of all targets indicating "Red" (A word "Green" displayed in green color). The encircled numbers indicate the stimulus conditions denoted in the factorial tree panel A, bottom line. **(C)** Actual stimulus condition realizations of the configuration presented in panel B.

Categorization - rule based vs. exemplar based processing

According to the exemplar-based models (Hintzman, 1986; Medin & Schaffer, 1978; Nosofsky, 1986), people form categories by storing individual exemplars in memory, while a decision process is based on similarity comparisons between target-items and stored exemplars. For example, an exemplar-based model represents categories *elephant* and *mouse* by storing all previously acquired exemplars that belong to these categories along with a category label. When a new animal is presented, it is compared to all exemplars belonging to the elephant category and mouse category. Finally, a decision whether the displayed animal is an elephant or a mouse is made based on the overall summed similarity of the displayed animal to each category.

An alternative to the similarity-based exemplar model is the idea that people use rules in order to classify objects. One rule-based approach assumes that people use decision boundaries for dividing a multidimensional psychological space into category-response regions (see e.g., Ashby & Gott, 1998; Ashby & Lee, 1991; Ashby & Maddox 1990; Ashby & Perrin, 1988). An observer can make independent decisions regarding a stimulus's value along each of its multiple dimensions and then combine these separate decisions to reach the final classification response. Take for an example, classification between mice and elephants based on two rules: an animal could be categorized as a mouse if its weight is less than 1 pound and its whiskers are longer than one inch; or it could be categorized as an elephant if its weight is greater than 1 pound and whiskers are shorter than one inch.

Research design: Fific, Little & Nosofsky (2010) examined rule-based processing using a stimulus set which varied on three factors now instantiated using cartoon lamps rather than mice and elephants (see the white boxes, Figure 4A). The factor of stimulus category indicates the stimulus category membership (category A or B) determined by the two properties: the width of the lamp base and the curvature of the lamp top. The factor of target type indicates the lamp features that could be used to correctly classify a lamp into category B. These features are termed the target features and can be used to uniquely identify the category B membership. For instance, to classify B category lamps, one can specify a criteria for each target feature against which each observed target features can be compared. The category B lamps have at least one target feature with a value lower than the criterion value for that feature. For example, in Figure 4C the three leftmost vertical lamps have all the least curved top element (x_1) of all the lamps ($x_1<x_2 < x_3$). All three leftmost lamps could be easily classified as B by using a simple disjoint classification rule: compare x value to the criterion value (represented as a vertical dotted line), and if it less than the criterion, classify the lamp as B.

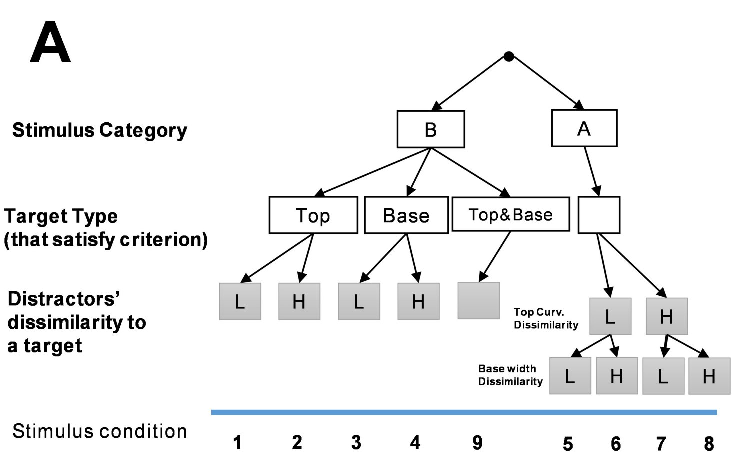
Depending on the type of the target feature, the three conditions are possible in which the top, base, or both features can be used for successful classification of a B member. The distractor features cannot be used to correctly categorize a B category lamp. These features are shared between A and B category members. In the category A, both lamp features are denoted as distractors so they can't be use to classify the category B membership when considered in isolation. If both features present in the lamp are distractors, then the lamp must be a member of A category (empty white box), which could be described as the conjoint rule that must be satisfied to make a correct classification of A lamp: check the values of each lamps feature and if both values are larger than their respective criteria values, then classify the lamp as A.

The third factor is related to the properties of the distractor feature. The factor of distractor dissimilarity indicates to what extent the distractor feature is similar to the nearest opposing category stimulus. The factor of distractor dissimilarity was manipulated at the low (L) and high (H) levels, depending on the stretching level for each process. The SFT addition to the original study is shown in Figure 4A, gray boxes. The diagnostic SFT conditions, displayed at the very bottom of the factorial tree, factorially combine the base width dissimilarity and the top curvature dissimilarity. The four conditions (HH, HL, LH, and LL) are also displayed in the Figure 4B providing another view of the factorial tree in Figure 4A obtained by renaming and combining the factors from the panel A. It is interesting to note that the two psychological dimensions of base and top dissimilarity correspond to the two continuous physical dimensions of base width and top curvature, respectively. The experimental condition values (1-9) shown in panel A are also displayed in panel B. The panel C shows the actual trials, defined in the panel B.

Defining a model: For multidimensional stimuli, an important consideration is how the individual features of objects are processed in order make a categorization decision. For example, in the exemplar-based approach, all of the stimulus features are weighted and summed into an overall similarity-strength measure. By contrast, one can also examine which feature source was processed first or second, whether processing of separate sources of information was conducted serially or in parallel, and whether the perceptual sources of information were treated independently. Fifić et al. (2010) introduced a class of rule-based models that can be used to assess different information-processing decision strategies employed by humans (see Griffiths, Blunden, & Little, 2016, this volume).

In the categorization task shown in Figure 4, subjects are presented with one of the lamp stimuli at the time and have to make decisions whether the lamp is a member of Category A or Category B.

The similarity-based exemplar model and the logical-rule based models make two distinct SIC predictions. The exemplar model predicts an S-shaped SIC function, with smaller negative part and then a positive part consistent with coactivity. The logical-rule models, however, also allow for parallel or serial processing (Fifić et al., 2010; Little et al., 2011; 2013), which predict differently-shaped SICs. More information about these models is discussed in Griffiths, Blunden, and Little (2016; this volume) and Cheng, Moneer, Christie, and Little (2016, this volume).



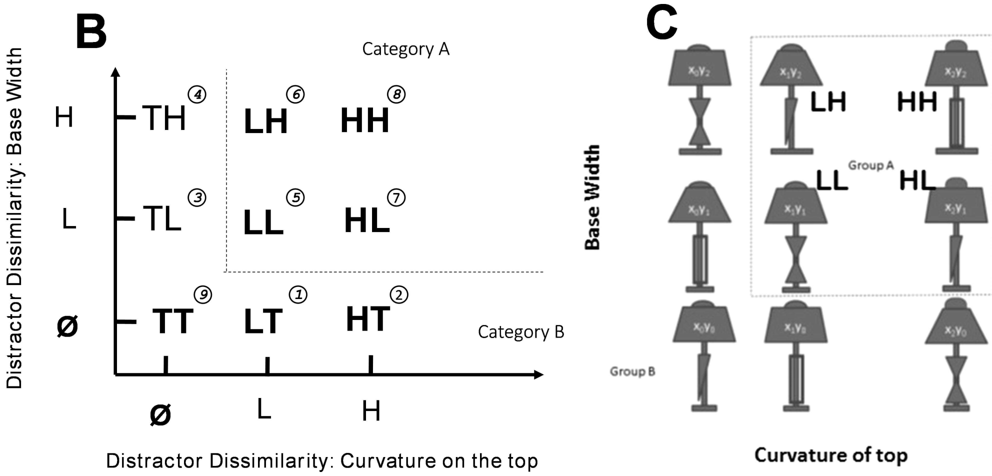


Figure 4: The categorization task - rule based vs. exemplar based processing. **(A)** The factorial tree represents the factors on left and the factor levels in boxes on right. The white boxes indicate the original study factorial design prior to the SFT modification. The shaded boxes indicate the added conditions after SFT process stretching has been applied. The double-factorial conditions are shown at the tree's bottom as another two-branch factorial tree made of two factors (distractor dissimilarity of the curvature on the top, and distractor dissimilarity of the base width). **(B)** The corresponding stimulus configuration space is defined by the two continuous dimensions: distractor dissimilarity of the curvature on the top, and distractor dissimilarity of the base width. The "TT" condition is so-called a stimulus redundant condition in which each of the both distinctive features available could be used to make B category classification. The distractor features are denoted as L and H as they are stretched across both dimensions. If a distractor feature is close to the linear decision bound (or an opposing category group member) this is defined as Low stretching effect. If a distractor feature is more distant, then this is defined as the High stretching effect. The double-factorial design is presented in the upper right corner (HH, HL, LH, and LL). The piecewise linear function separates the two response mapping regions: target present - which uses a disjoint rule (OR) to classify stimuli, and target absent – which uses a conjoint stimulus classification rule (AND). The encircled numbers indicate the stimulus conditions denoted in the factorial tree panel A, bottom line. **(C)** Actual stimulus condition realizations of the configuration presented in panel B.

Short Term Memory (STM) search

Short term memory (STM) is operationalized as a temporary storage for mental representations used for immediate cognitive operations. One of the important questions about the organization of STM regards the assumption about the processing order when searching stored representations.

In one STM task, subjects have to memorize a set of items and subsequently make decisions concerning whether or not a target item was member of that memorized set (e.g., Sternberg, 1966, 1969). An example of such a task using only two memorized items is presented in Figure 5C (Townsend & Fifić, 2004). The memorized sets consist of two three-letter words, in the consonant-vocal-consonant CVC form. Subjects would first learn the items in the memorized set and then, after a brief period, a target item was presented. If each memorized item word represents one source of information, one can ask how are these sources combined, and what is the processing when these items during STM search?

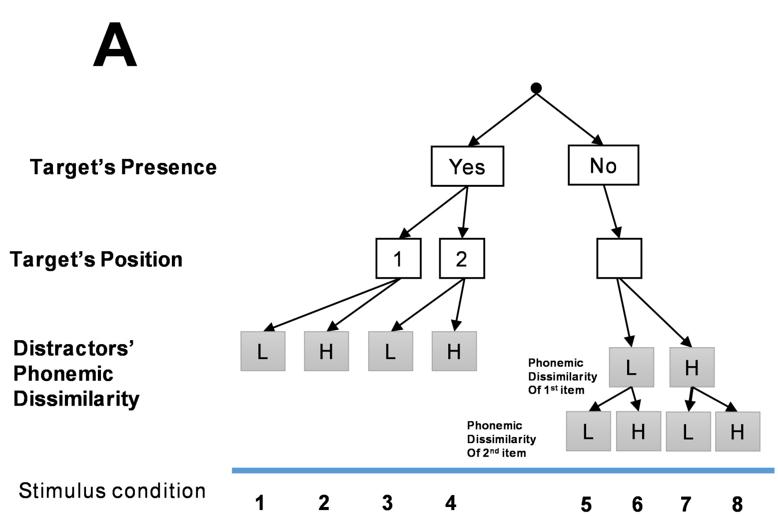
Research design: Townsend and Fifić's (2004) study had two factors, shown in the white boxes, Figure 5A. The factor of target's presence indicates whether a target item was presented in a memorized set or not (Yes or No). The factor of target position indicates the target position in the list of memorized items. (1= first place in the set, 2= second place in the set). The empty white box indicates that the target-absent condition in which both items must be searched to make a correct response. The items that are not identical to the target item are distractors.

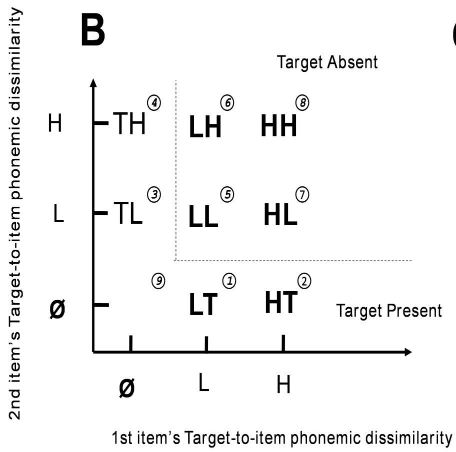
The SFT addition to the original study is shown in the gray boxes, Figure 5A. The added factor of interest was the distractors' phonemic item-to-target dissimilarity. The factor of distractor's phonemic item-to-target dissimilarity was binary manipulated at the two values, low (L) and high (H), depending on the stretching level for each distractor item. This is achieved by manipulating phonemic dissimilarity of all the items used in the task.All three-letter word items were grouped into a two phonemic categories: stimuli having nasal phonemes (L, M, N) and having fricative phonemes (F, S, V). For simplicity, the low item-to-target dissimilarity level was defined as the within-category relation. That is, both the memorized item and the target were selected from the same category (say LAM to

NAM or FAV to VAS). In contrast, the high item-to-target dissimilarity was defined as the between-category relation. That is both the memorized item and the target item were selected from the different categories (say LAM to FAV or NAM to VAS). The dissimilarity conditions of the distractors factorially combined with their position in the memorized set is shown in the Figure 5C.
The diagnostic SFT conditions are displayed at the very bottom of the factorial tree Figure 5A. These conditions are obtained by factorial combination of the phonemic dissimilarity of the first item in a memorized list, at the high and low levels, with the phonemic dissimilarity of the second item in a memorized list, at the high and low levels, but only for the target-absent responses. Panel 5B provides useful information of how the distractor item-to-target dissimilarity manipulation works. The two dimensions are defined on the continuous dimension scale of the item-to-target dissimilarity. All of the 8 experimental conditions from 5A can be mapped onto such a space. The target-absent conditions are positioned in the upper right corner and separated from the target-present conditions by a piecewise linear decision bound. The vertical bound requires analysis of the first item in the memorized set, comparison to the target, and a decision about whether this item is a target based on its dissimilarity level. If the dissimilarity level is zero (a match) or smaller than the critical decision value, then the item is identified as a target; otherwise, it is identified as a distractor. The same process also occurs (if necessary) for the second list item. The stretching effect is determined by the relative position of the stimulus condition relative to the imagined piecewise linear decision bound that separates the two response types. If a distractor item is close to the linear decision bound, then in practice this item phonemically sounds more like the target item. If a distractor item is distant from the linear decision bound, then in practice this items sounds more distinct from the target item.Conceptually, an exemplar-based approach could replace the linear decision bounds by the similarity-based functions between the items (Fifić et al., 2008; Nosofsky, et al., 2011).

<u>*Defining a model*</u>*:* In his seminal studies, Sternberg (1966; 1969) found that, in both the target-present and target-absent conditions, the mean RT increased linearly with the size of the memorized list and inferred that STM is scanned in a serial exhaustive fashion. One alternative approach proposed a parallel processing STM model in which all stored representations are analyzed simultaneously (Ratcliff, 1978). A third approach assumed that comparison of a recognition probe to all of the list items is pooled into a single, global memory strength variable (and is consequently commensurate with a coactive decision model; Nosofsky, Little, Donkin, & Fifić, 2011).

It is well-known that serial and parallel models of STM search cannot be distinguished at mean RT level due to model mimicry (e.g., Townsend, 1969; 1971; 1972; Townsend & Ashby, 1983, Chapter 14). Further attempts to differentiate processing using memorized set size functions have been unable to find conclusive evidence as to the underlying scanning architecture (Donkin & Nosofsky, 2012). SFT allows one to clearly delineate the predictions of each of the models at the RT distribution level (Townsend & Nozawa, 1995). The serial exhaustive, parallel exhaustive, and coactive models make distinct SIC signature predictions: serial exhaustive predicts the S-shaped SIC function, with equal positive and negative parts; the parallel exhaustive model predicts a strictly negative SIC function; and the coactive model predicts an S-shaped SIC function with a smaller initial negative component (Townsend & Fifić, 2004; see also Altieri, et al., Chapter 1, Figure 3).





C

Stimulus conditions		Memorized Items	Target
5	LL	MAL, NAM FAS, SAV	NAL VAF
7	HL	NAM, SAV FAS, NAM	VAS LAM
6	LH	NAL, FAS VAS, MAL	NAM FAV
8	HH	VAS , FAV MAN, NAL	NAL FAS
4	TL	**MAL**, NAM **FAS,** SAV	MAL **FAS**
3	TH	**NAM**, SAV **FAS**, NAM	NAM FAS
1	HT	NAL, **FAS** VAS, **MAL**	FAS MAL
2	LT	VAS , **FAV** MAN, **NAL**	FAV NAL

Figure 5: The short-term memory task. **(A)** The factorial tree represents the factors on left and the factor levels in boxes on right. The white boxes indicate the original study factorial design prior to the SFT modification. The shaded boxes indicate the added conditions after SFT process stretching has been applied. The double-factorial conditions are shown at the tree's bottom as another two-branch factorial tree made of two factors (distractor's phonemic dissimilarity of a distractor at the list position one, and the list position two). **(B)** The corresponding stimulus configuration space is defined by the two continuous dimensions: distractor phonemic dissimilarity of the distractor at the first list position, and at the second list position. The distractor features are denoted as L and H as they are stretched across both dimensions. The piecewise linear function separates the two response mapping regions: target present - which uses a disjoint rule (OR) to classify stimuli, and target absent – which uses a conjoint stimulus classification rule (AND). The double-factorial design is presented in the upper right corner (HH, HL, LH, and LL). The encircled numbers indicate the stimulus conditions denoted in the factorial tree panel A, bottom line. **(C)** Actual stimulus condition realizations of the configuration presented in panel B. The second column indicates the type of factorial conditions, as shown in panel B. The third column shows some examples of two memorized items, each consisting of three letters. The target item is show in the fourth column. By reading this three letter pseudo-words one can get a feeling of the phonemic dissimilarity effects, shown in the second column.

Integrative workspace

Figures 2-5A shows the standard factorial research design for four example tasks (displayed in the white boxes). Each tasks' factorial design is represented as a schematic factorial tree that branches across each factors' values. An immediate concern is to notice that all the original factorial designs cannot be characterized as full-factorial designs. When these designs are viewed as a factorial tree, one can see that the two main branches (left and the right from the main bifurcation point) are not symmetric (Figures 2-5 A, white boxes). In other words, the research design does not permit factorial combination of the all of the factors' values. They are defined as fractional factorial research design (FFD; e.g. Anderson & Whitcomb, 2015). Although helpful, FFD's do not allow for conducting the complete full factorial interaction test and certainly limits the number of applications. Nevertheless, researchers can conduct the set of simple factorial analysis by selecting the branches that makes full factorial designs. In practice, this is achieved by conditioning the analysis on a certain fixed values (see Fifić, 2016, for more details).

STEP 2: SFT design implementation (stretching)

Once the model(s) underlying the specific task are specified, one has to make a decision concerning how to proceed with the model testing using SFT. This step requires considering which processes within the model will be tested for their fundamental mental properties (processing order, stopping rule and process interdependency). Probing targeted processes within the model is achieved by stretching the duration of these processes through the manipulation of external factors. A researcher must establish a one-to-one relationship between the external factor and the underlying process, such that only one process is selectively affected by the single external factor. Furthermore, such an external manipulation should only affect the duration of the mental process of interest and should not incur speed accuracy trade-off or overall drop in accuracy.

The stretching effect has also been denoted as factor *saliency* or *discriminability* (Fifić, Townsend, & Eidels, 2008; Fifić & Townsend, 2010; Eidels et al., 2010; Eidels, Houpt, Altieri, Pei, & Townsend, 2011). A highly salient feature or processing unit (thus process) stands out among other processes and is associated with faster processing in the perceptual and cognitive literature (e.g. Triesman & Gelade, 1980).

The kinds of variables that could be used for process stretching depend on the type of research design and the task per se. In general, SFT-related research has brought out two types of stretching: The first type is (i) *external stretching*, which affects the processing speed of a target process due to manipulation of external stimulus' intensity. In the case of external stretching, the experimental variable directly affects the process of interest. There are different types of visual obstructions, such as stimulus masking, brightness (the redundant target task), size reduction, stimulus intensity (the Stroop task), and stimulus blurring, each of which could lead to slowing down or speeding up the processing rate of the process of interest. In Figure 2 C, each of the two stimulus dimensions (left and right dot) are defined on a continuous scale. The values of L and H are arbitrary satisfying the condition that the L value is associated with the lower brightness and the H value with the higher brightness. In Figure 3C, the color salience dimension is also stretched through the direct stretching method: as we go along the horizontal dimension the amount of redness is decreased.

The second stretching type is (ii) *internal stretching*, which affects processing speed of the target process by affecting its cognitive properties. To internally stretch a process, one can use a similarity function between item representations within the system. The concept of the item-similarity function can be instantiated as the distance in a Euclidian psychological space (Ashby & Maddox, 1994; Ashby, 2000; Nosofsky & Stanton, 2005; Fifić at al. 2010). For example, if a target item is closer in the similarity space to the non-target item(s) then discrimination of the target item should be slower. An alternative is to assume that it is the distance between an item and a decision boundary which determines the speed of processing (Ashby, & Maddox, 1994; Nosofsky & Stanton, 2005). In these examples, the manipulation by internal stretching does not directly affect the processing rate but rather it specifies the distance of the item in a space to other used items, which in turn affects processing rate.

For example, in the lamp classification task (Figure 4 C), the position of the target item is specified by two continuously-valued dimensions: the curvature of the top and the base width. The location of the target item specifies its relative position to the non-target items: the closer the distance, the more processing time is needed to classify the target item as a member of the target group. Thus, placing one item closer to the bound on one dimension should lead to creating a Low salience factor for that dimension. Placing an item farther away from the bound on one dimension should lead to creating a High salience factor for that dimension. In that manner, one can easily generate the four factorial conditions (HH, HL, LH, and LL).

In each of the task figures, the dotted lines are used to divide the regions of the stimulus space into two regions: one that contains the factorial manipulated targets and another that contains non-targets. The decision bound is presented in the form of the piecewise linear function, with each component parallel with one of the axes. To correctly categorize a stimulus, for any member of the target A category lamp stimuli in Figure 4B, one has to check two stimulus properties and compare them to two decision values.

It is important to note that the psychological decision bounds could be non-linear, or linear but also not orthogonal to the stimulus dimension axes. In such cases, the factorial placements of stimuli in the stimulus space would not necessarily lead to the desired level of stretching. Such a situation is defined as a violation of decisional separability (Ashby & Townsend, 1986). The test for a violation of selective influence could be used to indicate such violations and the inferences about the fundamental properties from such cases would be most likely invalid.

Although there is no trivial solution for detecting possible violations of decisional separability, several actions are possible: parametric models could be fitted to the RT (Fifić et al. 2010; Little et al., 2011; Little et al., 2013) to disentangle possible variants of the violations and verify the fundamental properties. Another less computer-intensive approach is to make the qualitative predictions about the diagnostic patterns not only limited to the double-factorial conditions (LL, LH, HL, and HH) but also to other stimulus conditions (the contrast category, see Fifić et al., 2010). The violation of the decisional separability and other concepts such as stimulus separability and independence (Ashby & Townsend, 1986; Maddox & Ashby, 1996; Kadlec, & Townsend, 1992; Silbert & Thomas, 2013) can be included within the SFT parametric approach (e.g., Blunden et al., 2015; Little et al., 2013). These approaches are discussed in Griffiths, Blunden, and Little (2016, this volume).

STEP 3: SFT Data collection and diagnosing mental architectures

The SFT approach uses RTs in each experimental condition collected over repeated trials. Selection of stretching effect for the experimental factor should permit error free performance of the subjects, although some moderate amount of error is permissible (see Fifić, Nosofsky, & Townsend, 2008, Appendix A). The MIC statistic uses mean RTs, while the SIC statistics uses distribution properties of RTs in the form of estimated survivor functions. It is important to check the possibility of violation of selective influence assumption before we proceed to the final diagnostic test.

The quickest test for the failure of the selective influence is to check the mean RT orderings:

$$RT_{LL} > (RT_{LH}, RT_{HL}) > RT_{HH} \qquad (4)$$

where each of the four terms represents the mean reaction time for one critical factorial condition. If the test fails and any of the inequalities is violated then the results cannot be interpreted straightforwardly, and one has to approach the result with the caution or provide a good explanation of why and how selective influence might fail in such situations.

Even more confidence in selective influence can be achieved by showing that the Equation 4 inequality holds when using survivor functions instead of using mean RTs. This can be achieved first by visual inspection:

$$S_{LL}(t) \geq [\ S_{LH}(t)\ ,\ S_{HL}(t)\ ] \geq S_{HH}(t), \quad \text{for all } t \qquad (5)$$

Statistical tests of the ordering can also be applied (Houpt & Townsend, 2010; Heathcote, Brown, Wagenmakers, & Eidels, 2010; Houpt, Blaha, McIntire, Havig, & Townsend, 2014; Yang, et al. 2014). The appropriate tests are the Kolmogorov-Smirnoff test or other nonparametric test (e.g., the cosphericity test, Kujala & Dzahafarov, 2008).

Statistical tests

The MIC values can be tested by using a corresponding ANOVA design. In addition to testing for the main effects to determine whether the stretching manipulation on each factor was effective, it is possible to test the hypothesis on whether or not an observed MIC value significantly departs from zero value.

There is a close relation between the results obtained using the MIC and SIC tests. This is because the MIC value is the integral of the SIC for all values of *t* (Townsend, 1991). This relationship can serve to provide a basic statistical hypothesis test for the visually observed SIC results, by using ANOVA on the MIC results (e.g. Fifić & Townsend, 2010; Fifić et al, 2008). Such inference could hold only if the selective influence assumptions also hold: the order of the stretching effects (L > H) and the ordering of the mean RTs and survivor functions are satisfied. The limitation of the MIC ANOVA test is that when attempting to distinguish between the parallel self-terminating and coactive models. Note that the MIC > 0 for both models so an MIC ANOVA test would fail to distinguish between the two SIC signatures (see Houpt, Blaha, McIntire, & Townsend, 2014). For this comparison, one needs to examine the SIC.

The second nonparametric test involves using bootstrapping (see Van Zandt, 2002, for details) to construct confidence intervals on the observed MIC values. Bootstrapping can also be used to generate the confidence regions around SIC function. The inference logic is similar to that of the MIC bootstrapping test: one has to check whether the function sufficiently departs from x-axis (zero value) such that for some times neither the SIC function nor the confidence interval around the SIC function captures zero value (see e.g., Yang, Little, & Hsu, 2014; Yang, Chang, & Wu, 2013).

The third non-parametric approach, specifically created to fit the SFT tools, was designed to test the maximal and minimal deviations in the SIC function shape using the mathematical properties of Brownian Bridge. The test is a generalization of the two-sample Kolmogorov-Smirnov test, and is available as an R-software package (Houpt, et al., 2013). See the chapter by Burns and Houpt (2016, this volume) for details.

Simple Double-Factorial design

The design is termed the double-factorial design (2 x 2: HH, HL, LH, and LL) when analyzing only two processes within a mental architecture. However, as we can see in Figures 2-5B, the double-factorial design is only a subset of the full-factorial design (depicted here as 3 x 3). A worthwhile question is whether all factorial conditions can be used to make even stronger conclusions in diagnostic fundamental properties?

First, one can apply further stretching manipulations to the factors, providing that one can find appropriate factor manipulations. Hence, one can produce both L, H, and an in-between value, M, such that the stretching effect are ordered: $S_L(t) \geq S_M(t) \geq S_H(t)$. Using these values one can create a 3 (H, M, L) x 3 (H, M, L) design. Out of the 3 x 3 design, it is possible to construct several simple double-factorial designs for example: (H, M) × (H, M), (M, L) × (M, L), (H, M) × (H, L), etc. Note that all conditions for the SFT analysis should be met in the simple tests as well. In addition, the simples tests must be conducted on the experimental conditions that are located on only one side of the imaginary decision bound, that separates the two different response regions (Figures 2-5, panel B). The first rationale is that stimulus conditions on the same side of the decision bound are usually (but not necessarily) fixed with respect to using only one stopping rule. So, combining the experimental conditions across the decision bound for the double factorial test, could combine response trials with different stopping rules used. That is comparable to testing a probabilistic mixtures model of fundamental properties within one cognitive system and introducing the new mixture parameters of relative proportions of terminating and exhaustive stopping rules. The second rationale is that the stretching effect is monotonically related to the distance from a stimulus condition to the decision bound, and combining response trials across different sides of the decision bound could lead to combining different stretching effects that may not satisfy the conditional stretching effect order at the marginal level ($RT_{low} > RT_{high}$).

Once the factorial conditions are collected (on one side of the decision bound) then the interpretation of the simple SFT tests are straightforward. Providing that the system doesn't change its underlying architecture across stretching levels, all of the simple double-factorial designs should predict the same outcome. For example, all the simple double-factorial tests should indicate the same type of MIC or SIC signatures. If different SFT results are found for different factorial combinations then this would indicate that the mental architectures changed for different levels of stimulus saliency. Such a result could be a strong indicator that there are some other properties used in the study that are not controlled by a researcher.

Summary

In the past decades, cognitive psychologists have laboriously tested models of cognitive operations. The models have varied in their scope with domain general models on one side and domain specific models on the other side. The veridicality of the models has usually been judged by whether or not the model can fit some pattern of data, typically using likelihood measures. The most advanced approaches utilize Bayesian inference to find the most probable model given the observed data among several candidate models. A disadvantage of the Bayesian approach is that it requires adopting (sometimes numerous) assumptions about the distribution of parameters and the distribution of the data. Similar but not identical approaches in model selection are based on maximizing likelihood functions of the observed data given the candidate models. This approach depends on intensive data simulation and parameter value search. By contrast, SFT is a parameter-free approach which uses the double-factorial design seeking data patterns which could rule out entire model classes. These model classes are defined by the fundamental properties of processing order, stopping rule, process interdependency, and processing capacity.

The qualitative forms of the SIC and MIC signatures are invariant across cognitive domains. Assuming the same organization of mental processes, the same SIC signature should be observed in the domain of visual search or in categorization. Likewise, SIC and MIC are invariant with regard to the distribution of stochastic variables. The same SIC signature should hold for two, e.g., serial exhaustive processing systems, assuming different distributions of the random variable completion times (e.g., exponential, gamma; but see the chapter by Harding et al., 2016, this volume).

SFT cannot be seen as a substitute for the model selection methods such as Bayesian inference. Rather than comparing and selecting full processing models, SFT can be used to test the proposed components of the models of interest and to verify whether or not the components are valid. For example, consider a scenario in which a complex cognitive model has been tested using SFT. If the cognitive model fails on the SFT test for the proposed processing order, then the model is immediately falsified. As a consequence, it is not reasonable to include this model in the set of plausible models. Consider another hypothetical example in which one compares three cognitive models using Bayesian model selection. Assume that in truth all three of the models are incorrect with respect to one of the assumed fundamental properties such that the compared models would be falsified by the SFT tests. Although this failure would reduce the likelihood of the model, one could still erroneously conclude that one of the three models was the correct model. While it is true that all models are false, some models can be immediately falsified as not possessing the critical processing component, by using SFT.

Over the course of the last decades the number of publications utilizing SFT has steadily soared. To the initiate the requirements of using, SFT may seem somewhat daunting at first glance. The current tutorial, as well as this book, aims at providing the integral overview of how to master the mental process stretching within SFT and how to apply the SFT to different cognitive tasks and models.

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